Physics of Few-Body Quantum Systems and Analytic Methods in Nuclear Physics

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History

Two-body quantum systems → Schrödinger and Lippmann-Schwinger equations.

Three-body quantum systems → Asymptotics unknown, Lippmann-Schwinger equation non-Fredholmian

L.D.Faddeev → reconstructed Lippmann-Schwinger equation and derived correct 3-body Faddeev equations (1960)

O.A.Yakubovsky → Faddeev-Yakubovsky equations for 4-body and arbitrary body systems

S.P.Merkuriev, S.L.Yakovlev et al. → account of Coulomb interaction

Applications

- 3-body nuclear systems, simplified NN potentials
- 3-body nuclear systems, realistic NN potentials
- 3- and 4-body systems, realistic NN and 3N forces.
  Systems with hyperons. 3- and 4-cluster systems
  (YNN, YNNN, ^{6}Li, ^{9}Be, ^{12}C, ^{16}O)
- n^{4}He-scattering (FY-equations, R.Lazauskas, 2018)
- Efimov states (theory – 1970, experiment – 2006)

A new field has emerged – few-body physics.

Faddeev medal (Vitaly Efimov and Rudolf Grimm, 2018)

Other methods

Few-Body Activity at SINP
I. Nuclear physics

- 3N,YNN, NN$\bar{N}$ (Yu.V.Orlov, L.B.)

- First calculations of ANCs for an α-particle using 4-body Faddeev-Yakubovsky equations (A.G.Baryshnikov, L.B.)

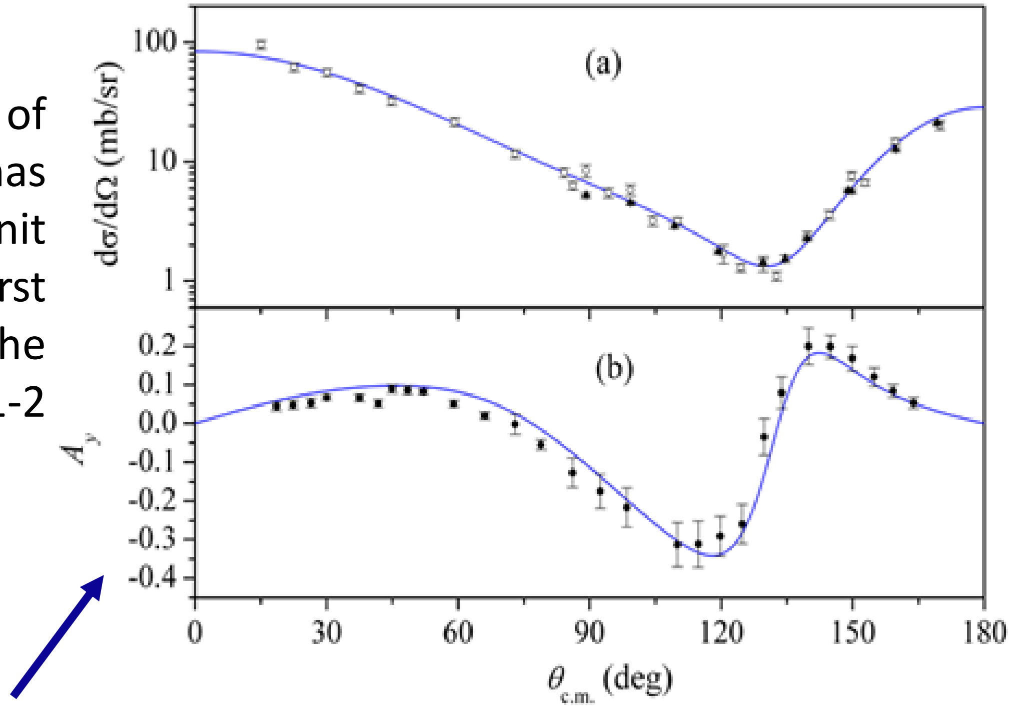
- ANCs for ^{6}Li, Faddeev equations, 3-body α+p+n model (L.B., V.I.Kukulin, D.A.Savin)

Solving few-body scattering problems via the wave-packet continuum discretization

V.I. Kukulin, V.N. Pomerantsev, O.A. Rubtsova , Ann. Phys. 2015; Comp. Phys. Commun. 2016.

A new path for solving few-body scattering problems is proposed. The main idea is the formulation of scattering problems in a discrete basis of stationary wave packets, which makes it possible to describe scattering problems just as in problems in discrete spectrum. This leads to the replacement of the complex multidimensional singular integral equations with linear algebraic equations.

As the next step, the practical solution of the Faddeev equations for 3N system has been realized on a graphics processing unit (GPU) on a personal computer for the first time. This algorithm sharply accelerate the time for solving such problems (by 1-2 orders of magnitude).



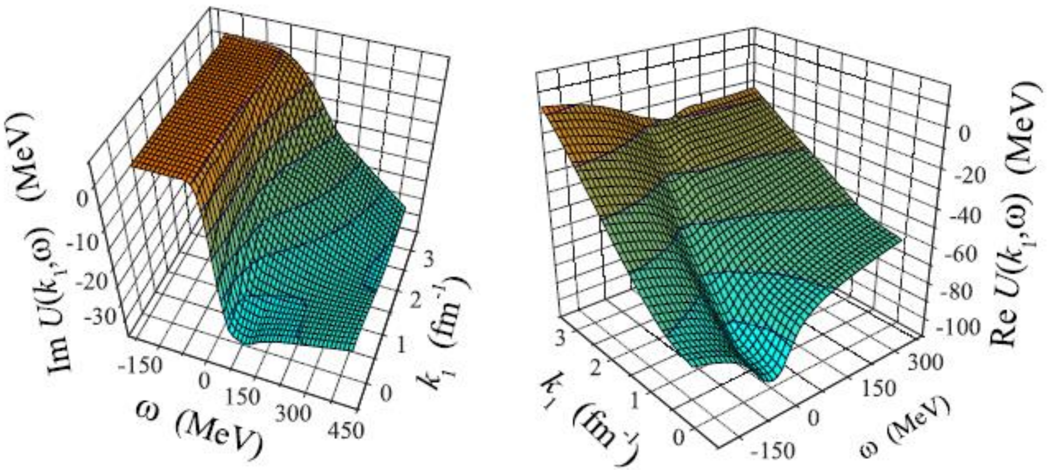
Differential cross section (a) and vector analyzing power (b) for elastic nd-scattering at En = 35 MeV found in WP technique on GPU (solid curves) in comparison with experimental data. Here fully realistic Nijmegen NN potential has been used.

Wave-packet discretization for nuclear matter calculations

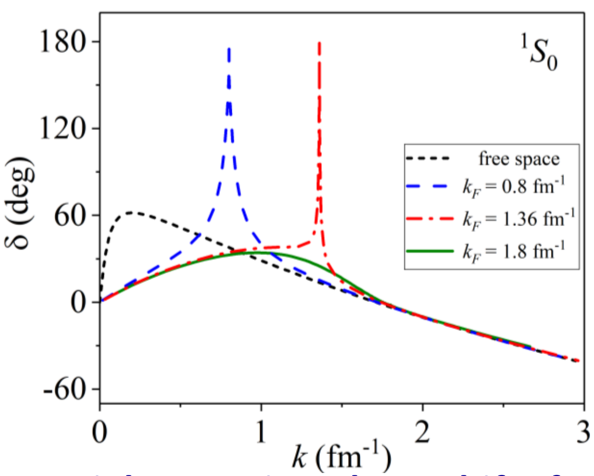
O.A. Rubtsova, V.I. Kukulin, V.N. Pomerantsev & H. Müther, PRC 2016, PRC 2017, PRC 2021.

Wave-packet approach has been also used for solving equations in nuclear medium within the self-consistent BHF scheme.

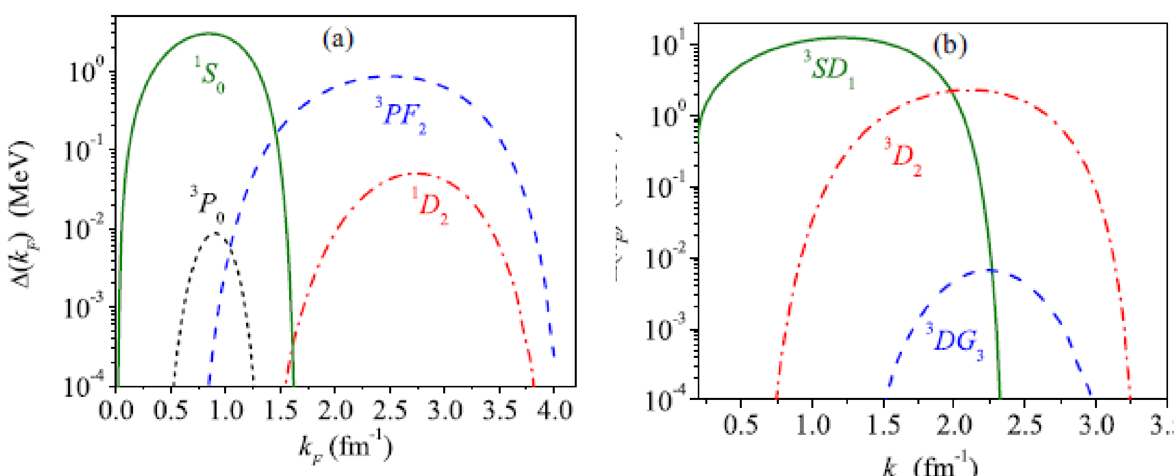
The method has been extended to an account of 2p-2h correlations when the pairing instability may occur. A new technique for finding the in-medium T-matrix and paring gaps has been suggested for such a case.



Self-energy found via the WP technique within the Brueckner-Hartree-Fock formalism



Partial scattering phase shifts for $^1S_0$ NN channel at different densities of nuclear medium

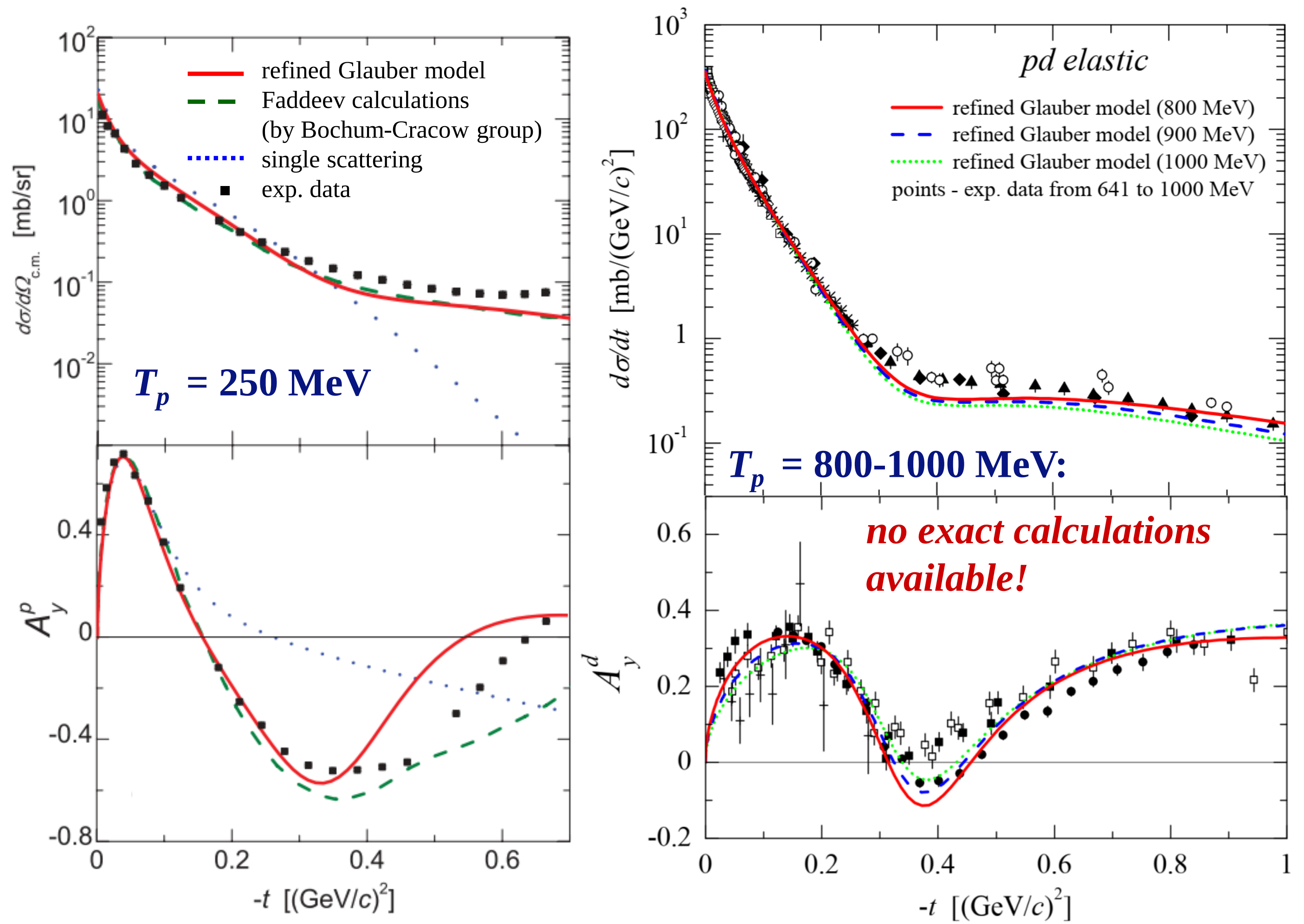


Pairing gaps for different partial NN channels in neutron matter (a) and symmetric nuclear matter (a and b).

Refined Glauber model for spin observables in intermediate-energy *pd* elastic scattering

- Elastic proton-deuteron scattering gives information about the nature of 2*N*- and 3*N*-interactions.
- Recent high-precision data: COSY (Juelich), RCNP (Osaka), Nuclotron (Dubna).
  New measurements of pd elastic spin observables are planned at NICA-SPD (Dubna).
- Refined Glauber model with full account of spin degrees of freedom describes not only unpolarised cross sections, but also sensitive spin observables at small momentum transfers $|t| < 0.2\text{-}0.3\ (\mathrm{GeV}/c)^2$.
- The model can be used for antiproton-deuteron scattering as well.

Results for differential cross section and proton and deuteron vector analysing powers: comparison with exact Faddeev calculations and exp. data



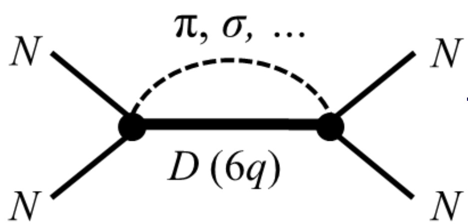
[M.N. Platonova & V.I.Kukulin, PRC 81, 014004 (2010); EPJA 56, 132 (2020); C. Fritzsch et al., PLB 784, 277 (2018)]

Dibaryon concept for nuclear forces

V.I. Kukulin, V.N. Pomerantsev, I.T. Obukhovsky et al., J. Phys. G 2001, Int. J. Mod. Phys. 2002, Ann. Phys. 2010

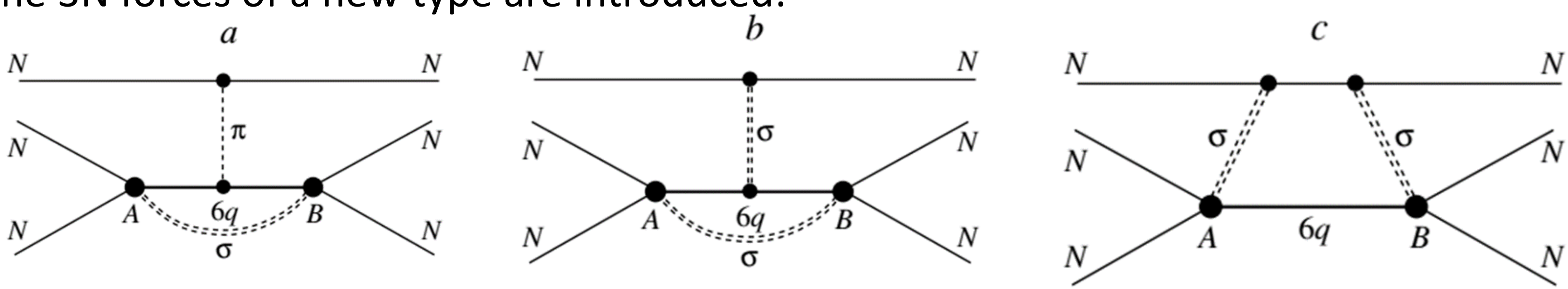
The theoretical group in SINP MSU has a long-term experience in developing the NN interactions taking into account non-nucleonic degrees of freedom and 6q symmetries, such as the deep NN potential with a forbidden state (V.G. Neudatchin et al. 1975) and Moscow NN potential (V.I. Kukulin et al. 1985).

Later on, the new mechanism for the short range NN interaction has been suggested:



the **s-**channel dibaryon exchange

This mechanism replaces the conventional *t*-channel scalar (σ-) exchange between two nucleons by the *s*-channel exchange with 6q state dressed with the σ-meson field, also the 3N forces of a new type are introduced:



The dibaryon model can be viewed as a bridge between the traditional nuclear physics dealing with the nucleon and meson degrees of freedom and the QCD operating with quarks and gluons. This concept should have the far-going consequences for hadronic and nuclear physics as well as astrophysics.

Properties of ^{3}H and ^{3}He from 3N calculations with the dibaryon model

Model	E, MeV	P_D, %	$P_{S'}$, %	$P_{6qN}(P_{\text{in}})$, %
^{3}H				
DBM(I) $g = 9.577^{a)}$	−8.482	6.87	0.67	10.99
DBM(II) $g = 8.673^{a)}$	−8.481	7.08	0.68	7.39
AV18 + UIX[b)]	−8.48	9.3	1.05	–
^{3}He				
DBM(I)	−7.772	6.85	0.74	10.80
DBM(II)	−7.789	7.06	0.75	7.26
AV18 + UIX[(b)]	−7.76	9.25	1.24	–

a) The values of the σNN coupling constant *g* in ^{3}H calculations have been chosen to reproduce the exact binding energy of the ^{3}H nucleus. The calculations for ^{3}He have been carried out without any free parameters.
b) S. C. Pieper et al., Phys. Rev. C **64**, 014001 (2001).

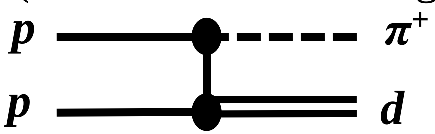
The model allowed to reproduce quite accurately the Coulomb difference:

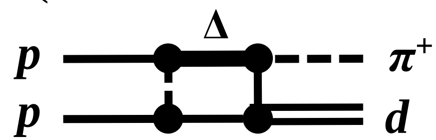
$$\Delta E_{\text{Coul}} = 754 \text{ keV} \quad \left(\Delta E_{\text{Coul}}^{\text{exp}} = 764 \text{ keV}\right)$$

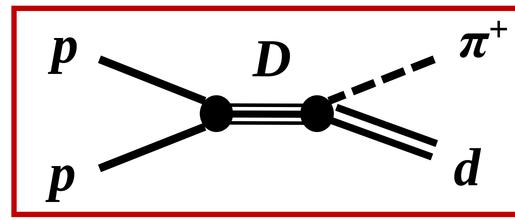
Dibaryon model for single-pion production in *pp* collisions

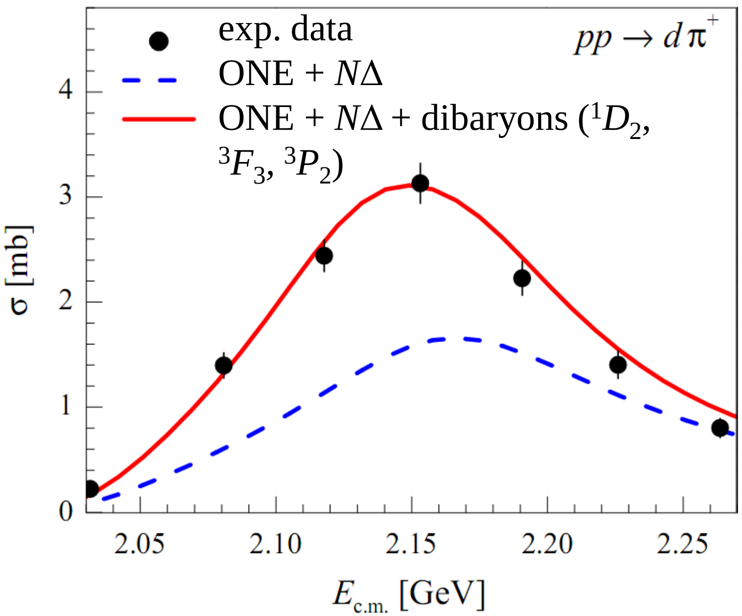
- A series of isovector dibaryon resonances in *pp* channels $^1D_2$, $^3F_3$, $^1G_4$, etc., have been known since 1970s from experiments (I.P. Auer et al.) and PWA (N. Hoshizaki; R. Arndt et al.).
- New ANKE-COSY data on $pp \to pp\,(^1S_0) + \pi^0$ revealed also $^3P_2$ and $^3P_0$ resonances with a mass of 2200 MeV (V. Komarov et al., PRC 93, 065206 (2016)).
- **Model for $pp \to d\pi^+$ proposed at SINP MSU:**

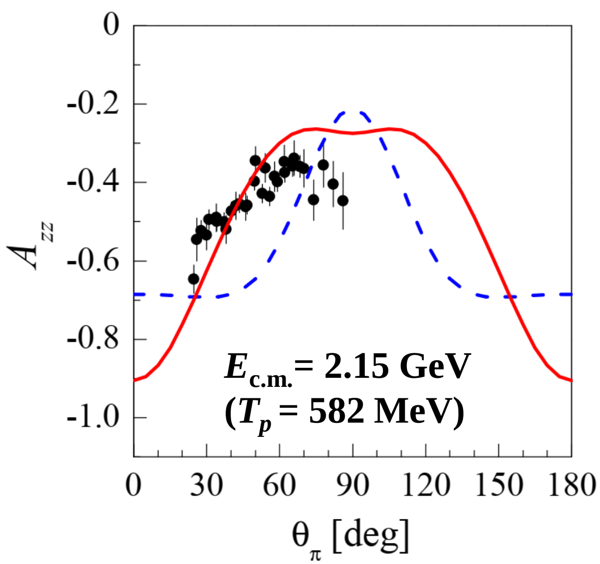
ONE (one-nucleon exchange) + $N\Delta$ (N+Δ intermediate state) + **New dibaryon mechanism**











Total cross section and pp spin-correlation parameter at peak energy

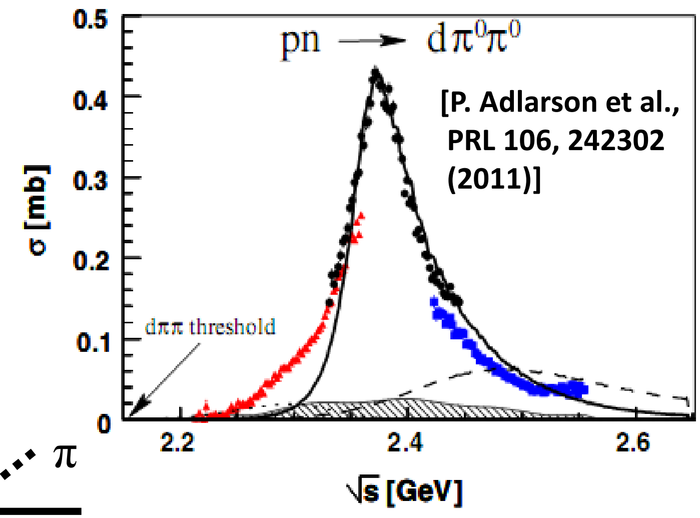
[M.N. Platonova & V.I.Kukulin, NPA 946, 117 (2016); PRD 94, 054039 (2016)]

The model with intermediate dibaryon resonances allows for a much better description of the reaction $pp \to d\pi^+$ than traditional models including only baryon resonances (mainly Δ-isobar), both for unpolarised cross sections and spin observables.

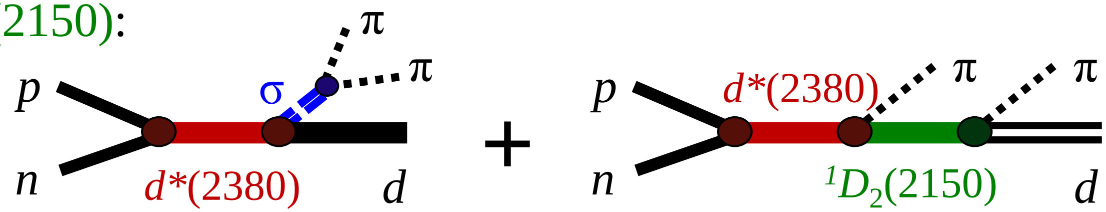
Dibaryon model for double-pion production in *pn* collisions and explanation of the ABC effect

- The Abashian-Booth-Crowe (ABC) effect – an anomalous near-threshold enhancement in double-pion production in pn, pd and dd collisions (first observed in Berkeley in 1960).

- New exclusive experiments of the WASA@COSY Collaboration: **discovery of the dibaryon resonance** $\boldsymbol{d^*}$ **(*M* = 2380 MeV, *Γ* = 70 MeV)** in $pn \rightarrow d\,\pi^0\pi^0$.



- **Model proposed at SINP MSU:** emission of two pions from the $d^*(2380)$ dibaryon via a scalar σ meson (from the meson cloud of d^*) and an intermediate dibaryon $^1D_2(2150)$:

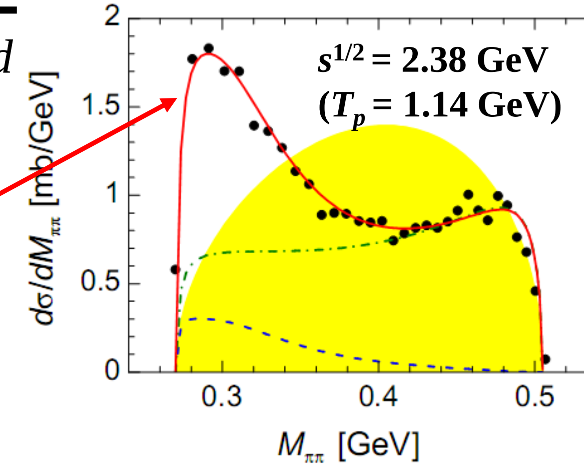


The ππ invariant-mass spectrum at the d* peak:

- ● exp. data, WASA@COSY
- phase space
- $d^* \rightarrow d + \sigma$
- $d^* \rightarrow {}^1D_2(2150) + \pi$
- full calculation

ABC effect – signal of the σ-meson production!

[M.N. Platonova & V.I. Kukulin, PRC 87, 025202 (2013)]

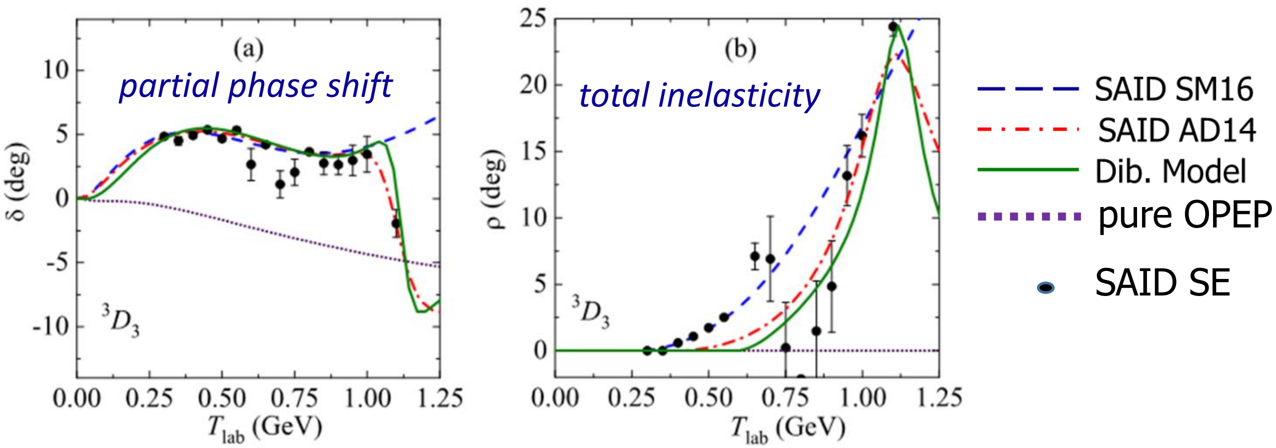


Production of scalar σ-mesons in *NN* collisions supports the new dibaryon mechanism of *NN* interaction and, in general, the dibaryon concept of the nuclear force suggested at SINP MSU.

Elastic scattering amplitudes in partial NN channels

V.I. Kukulin, V.N. Pomerantsev, M.N. Platonova, O.A. Rubtsova & H.Clement, Phys. Lett. B 2020, EPJA 2020.

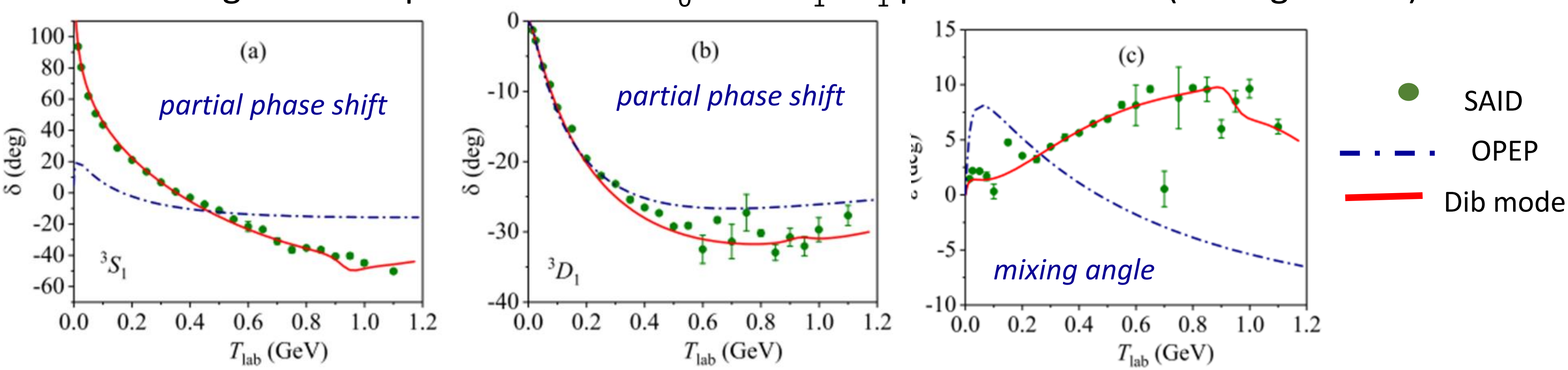
Elastic scattering phase shifts and total inelasticities for the *NN* coupled channels $^3D_3$-$^3G_3$ are explained with an account of d*(2380) resonance found in 2014.



Resonance parameters found within dibaryon model compared to **exp. values**:

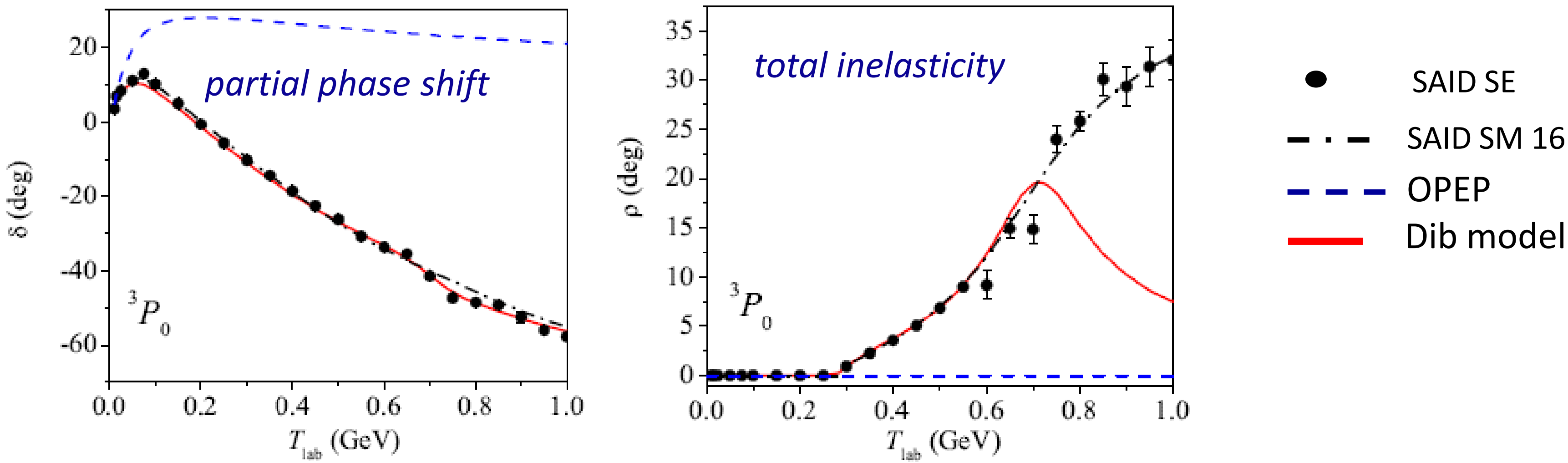
M_{th}(d*)=2.376 GeV **[2.38 GeV]** , Γ_{th}(d*)=0.084 GeV **[0.08±0.01 GeV]**

Very recently a new experimental evidences for two dibaryon resonances near the NN*(1440) - threshold have been found by WASA-COSY group. Couplings with these resonances govern NN phase shifts in $^1S_0$ and $^3S_1$-$^3D_1$ partial channels (see Fig. below).



$^3P_0$ NN channel and the near-threshold pion production

O.A. Rubtsova, V.I. Kukulin, M.N. Platonova, PRD 102,114040 (2020).



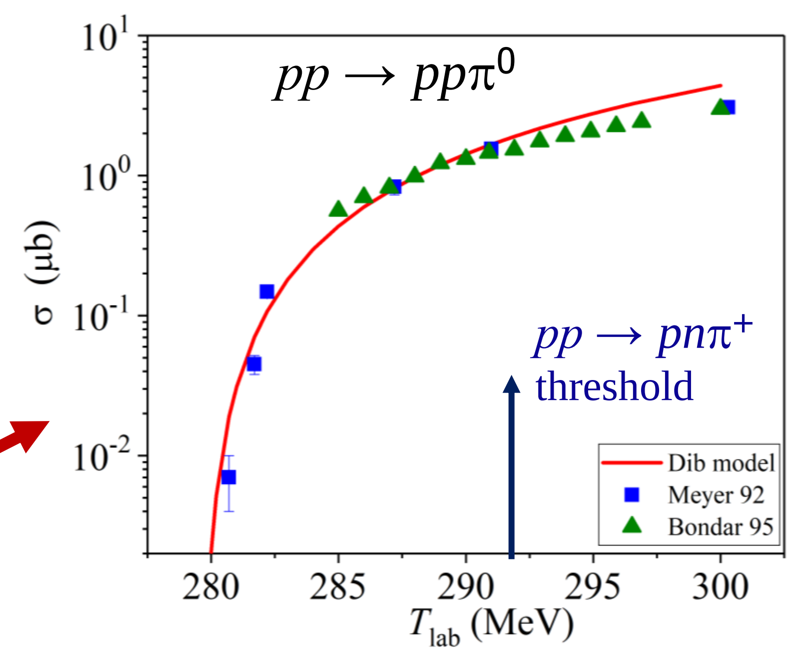
Exp. position for $^3P_0$ dibaryon:

M_R=2201 ± 5 MeV, Γ = 91 ± 12 MeV

Resonance position found within the dib. model:

M_{th}= 2200 MeV, Γ_{th}= 99 MeV

The near-threshold cross section of the neutral-pion production is described quantitatively via the resonance mechanism with $^3P_0$ dibaryon found experimentally in 2016.



II. Atomic physics

The scattering theory in atomic and molecular physics has significant differences from the scattering theory in nuclear physics.

- Long-range Coulomb interaction instead of a short-range nuclear one.
- Systems with particles of very different mass

These specific and important points were considered in a series of works by Yu.V. Popov with his colleagues (K.A.Kuzakov, et al.).

- It was shown theoretically and experimentally confirmed that taking into account the interaction of electrons in the final state of the reaction (e, 2e) noticeably affects their angular distribution.
- The divergences of the terms of the multiple scattering series in a system of 3 charged particles associated with the Coulomb interaction were investigated, and ways of dealing with these divergences were developed.
- It was pointed out the specificity of the Faddeev equations in the case when the mass of one of the particles tends to infinity.

- Electron transfer reactions, Faddeev equations, K-matrix approach (G.V.Avakov, L.B., A.M.Mukhamedzhanov)

- **Exotic atoms (muons, pions, antiprotons).**

Description of processes related to the formation and subsequent collisions of excited exotic systems with atoms and molecules of the medium, especially hydrogen and helium. (G.Ya.Korenman, S.N.Yudin, V.P.Popov, V.N.Pomerantsev)

Analytical Methods

- The analytic methods are based on the assumption that amplitudes of physical processes are analytic functions of their kinematic variables (energy, scattering angle, etc.) (microcausality principle).
- Various non-trivial relations follow from the property of analyticity of a function, the most important of which is the Cauchy theorem. From it follows the dispersion relations connecting the real and imaginary parts of the function, in our case, the amplitude of the process. An important property of an analytic function is the possibility of its analytic continuation from one domain of variables to another.

- The use of the property of analyticity of amplitudes allows one to obtain important relations linking various physical quantities without specifying the dynamics of processes.
- These methods were originally applied in quantum field theory and the theory of elementary particles, but later they became increasingly used in the nonrelativistic theory of nuclear and atomic collisions. They do not replace the methods based on the Schrödinger formalism of wave functions and interaction potentials, but supplement them.
- These methods make extensive use of the unitarity property of the S matrix and the formalism of Feynman diagrams.

Analytical Methods in Low and Medium Energy Nuclear Physics at SINP

- The use of these methods at SINP has a long history dating back to I.S.Shapiro. He proposed to approximate the amplitudes of nuclear reactions by the sum of contributions from the singular points of the amplitude closest to the physical region of the reaction, corresponding to the simplest Feynman diagrams that contribute to the reaction cross sections.

- When considering processes at low and medium energies, it was natural to use nonrelativistic kinematics and, in particular, the apparatus of nonrelativistic FDs.

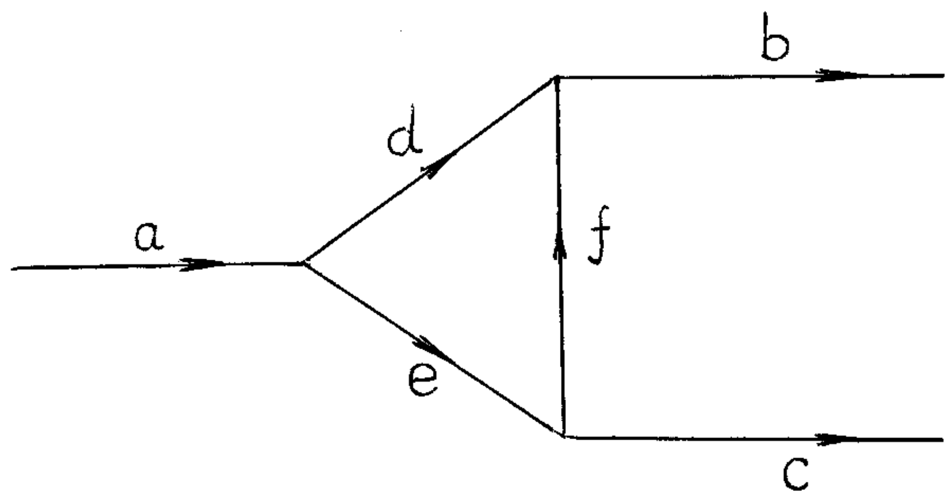
- In this regard, at SINP, the formalism of nonrelativistic FDs was developed, which made it possible to noticeably advance in calculating the amplitudes of these diagrams and the positions of their singular points in comparison with the results for relativistic FDs (L.B., E.I. Dolinsky, V.S. Popov).
- The positions of the singularities were found for an arbitrary square diagram, a "square with a diagonal" diagram, and for the partial amplitudes of a series of multiple rescattering in a system of three particles of arbitrary mass. Explicit expressions were obtained in general form for the amplitudes of the triangular and square diagrams (in the relativistic case, only for special cases of the triangular diagram).

- A dispersive approach was developed to construct effective potentials between composite systems (A.N.Safronov). Methods for taking into account Coulomb effects in dispersion relations have been developed (A.N. Safronov). The change in the behavior of FDs near their singularities in the presence of the Coulomb interaction has been established (A.M. Mukhamedzhanov).

- The DOM has been significantly developed, within which the optical potential is considered as an analytical function of the collision energy, and DSs are written for it (E.A. Romanovsky, O.V. Bespalova). Achievements in the description of the 1-particle structure of nuclei within the framework of this model are reflected in the thesis of O.V. Bespalova (September 2020).

- An analytical continuation of the integral equations of the theory of scattering onto a nonphysical sheet was carried out (Yu.V. Orlov).

- Based on the use of the analytic properties of the form factor of the vertex $a \rightarrow b + c$, a non-standard (anomalous) asymptotics of the wave function of a system of 3 or more particles in a two-particle channel was found (L.B.).


a
b
c
d
e
f

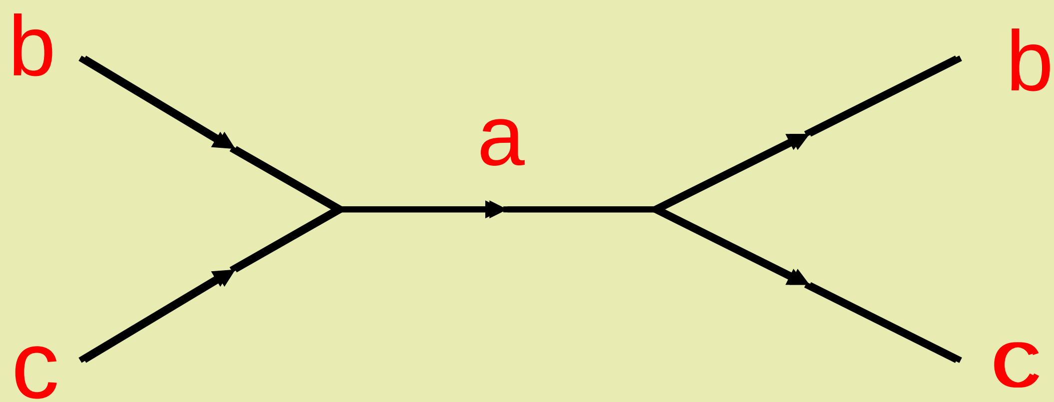
- Analytic methods have been successfully applied to determine ANC values from experimental data.

- ANC is the coefficient in the asymptotics of the wave function of a nucleus in a 2-fragment channel: $a \to b + c$. (overlap integral). In terms of ANCs, cross sections for peripheral nuclear reactions are parameterized when the reaction occurs at large distances between participating fragments. Peripheral reactions include reactions with charged particles at low energies (Coulomb barrier).

- The most important example of such reactions are astrophysical nuclear reactions, a significant part of which cannot be measured in laboratory conditions due to the smallness of the cross sections, which exponentially decrease as the energy tends to zero (in the core of the Sun E = 1.4 keV). Hence the importance of theoretical predictions and ANC.

- For the first time, the role of ANC in astrophysical nuclear reactions was indicated in the works of A.M.Mukhamedzhanov (radiation capture).
- ANCs are important nuclear characteristics not only due to their role in astrophysics.

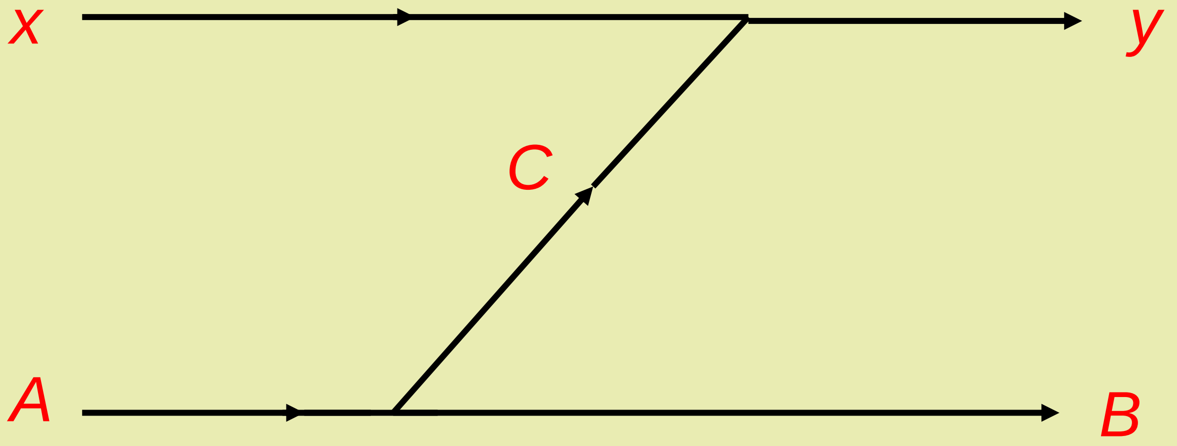
- ANC for $a \rightarrow b + c$ are expressed through the residue of the elastic $b + c$-scattering amplitude at the pole corresponding to the bound state $b + c$, lying in the non-physical region at negative energy ($E = -\varepsilon$). Therefore, the idea arose - to find ANC by analytic continuation of the experimental data of bc-scattering to the pole $E = -\varepsilon$.
- In the work Phys. Rev. C48, 2390 (L.B., V.I.Kukulin, D.A. Savin et al.) by this method ANC was determined for $^{6}Li \rightarrow \alpha + d$. This important coefficient determines the cross-section of the radiative capture
- $^{4}He(d, \gamma)^{6}Li$. The ANC value obtained in this work is now recognized as a reference.



b
a
b
c
c

- By this method at SINP ANCs have been recently determined for bound and resonant states of a number of nuclei ($^{7,8}Be$, ^{13}C, $^{16,17}O$) (Yu.V. Orlov, L.B., D.A. Savin).

- Another possible method for determining ANCs from the experiment is the analytic continuation of the DCS of transfer reactions to the pole in the scattering angle. Such work is now being carried out at OFAYA (L.B., D.A. Savin).


x
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C
A
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Thank you for patience