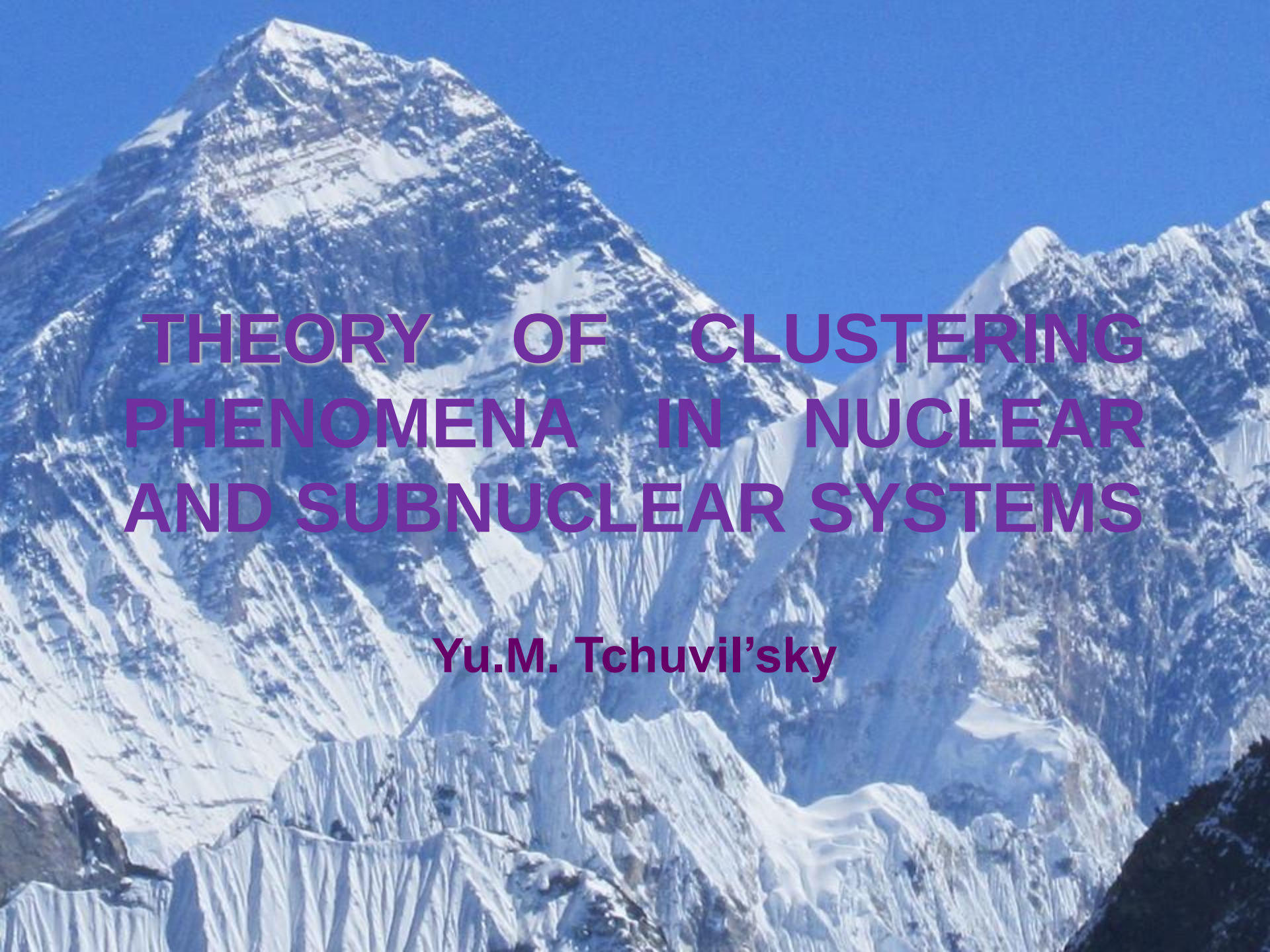
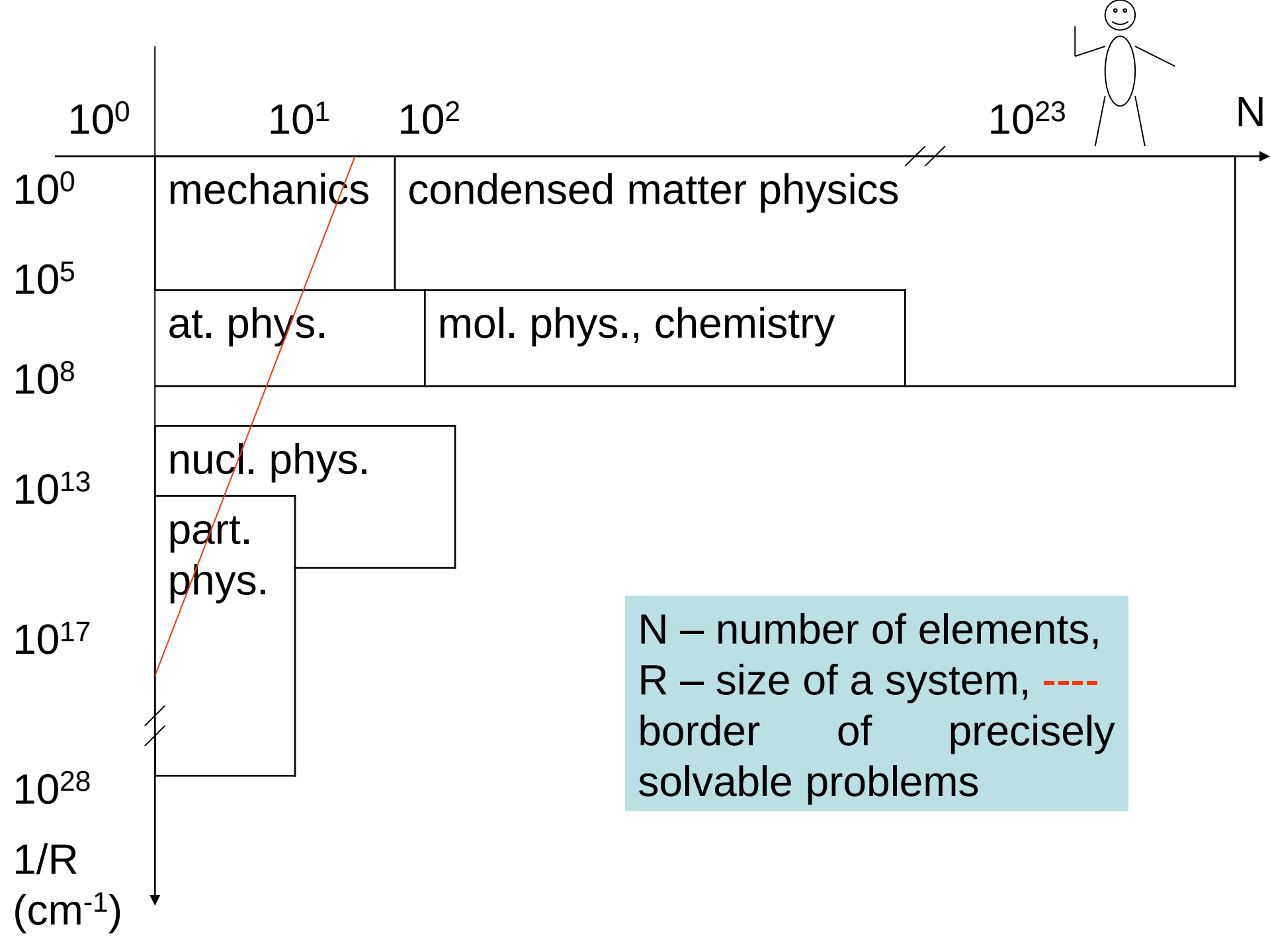


THEORY OF CLUSTERING PHENOMENA IN NUCLEAR AND SUBNUCLEAR SYSTEMS

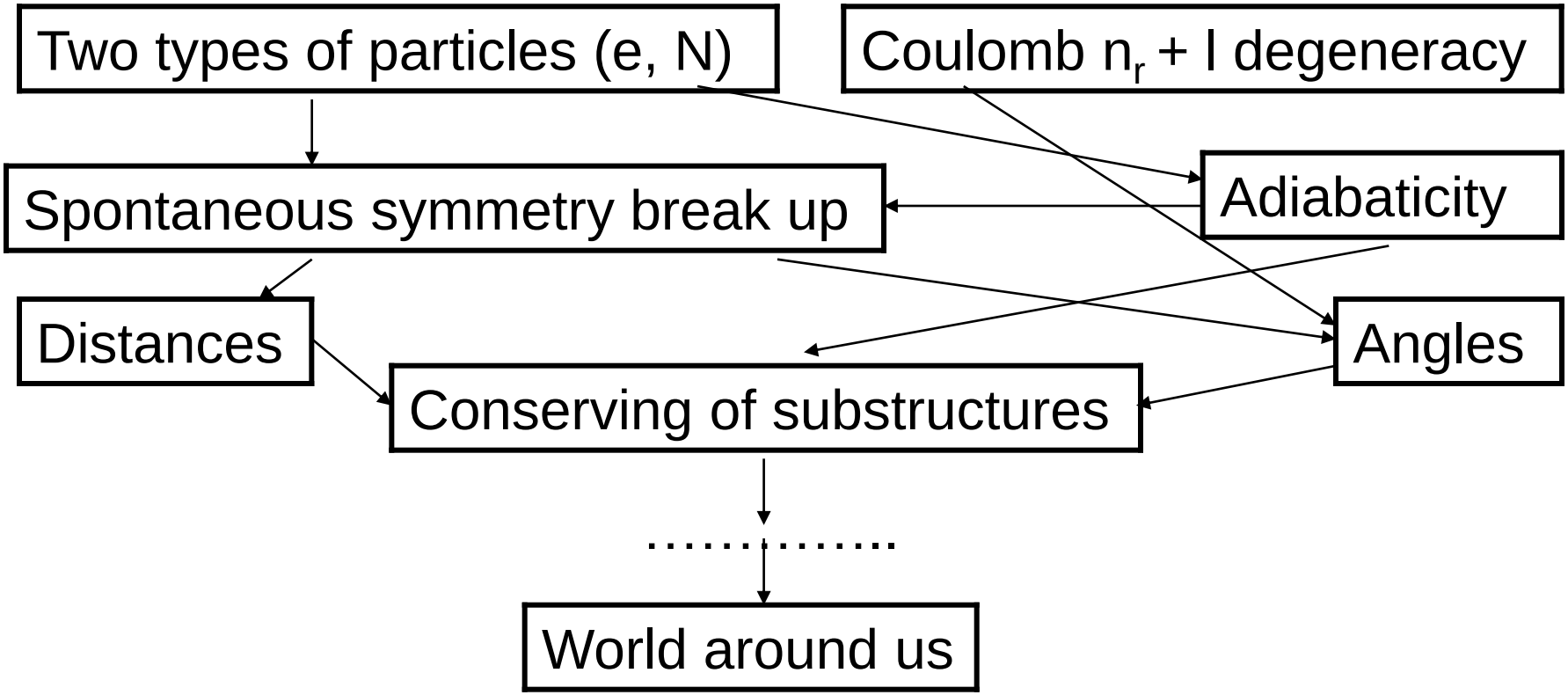
Yu.M. Tchuvil'sky


10^0
10^1
10^2
10^{23}
N
10^0
10^5
10^8
10^{13}
10^{17}
10^{28}
1/R
(cm^{-1})
mechanics
condensed matter physics
at. phys.
mol. phys., chemistry
nucl. phys.
part. phys.
N – number of elements,
R – size of a system, ---- border of precisely solvable problems

PROBLEM OF STRUCTURING

How to state the problem of substructures (clusters) in two- (or few-) cluster system taking into account indistingvishibility of identical fermions and the strong nuclear interaction?

STRUCTURING OF MOLECULES



HISTORY OF CLUSTERING RESEARCHES

Beginning:
Resonating Group Model, RGM (Wheeler, 1937).

. . . . . .

A unified theory of the nucleus (Wildermuth and Tang, 1977).

. . . . . .

Clustering in the shell model and its adoption to the decay and reaction theory. Spectroscopic factors. (Mang,1957 – alpha-decay; Balashov, Neudatchin, Smirnov, Yudin, 1959 – transfer and knock-out reactions).

Multiple contributions:

Smirnov et al – shell-model algebra.

Neudatchin et al – cluster-cluster potential with forbidden states.

Kukulin et al – multicluster dynamical model.

Zelenskaya et al – cluster transfer reactions.

. . . . . .

COMPOSITE PARTICLE INTERACTION IN THE RGM. ONE-CHANNEL PROBLEM

The wave function (WF) of the resonating group model (Wheeler, 1937) is chosen in the form:

$$\Psi_{A_1+A_2} = \hat{A}\{\Psi_{A_1}\Psi_{A_2}\varphi(\vec{\rho})\},$$

where

$$\hat{\mathrm{A}} = \begin{pmatrix} A \\ A_1 \end{pmatrix}^{-1/2} \left(1 + \sum_P (-1)^p \hat{P}\right)$$

The A-fermion Schrödinger equation

$$\hat{H}\Psi_{A_1+A_2} = E\Psi_{A_1+A_2}, \quad \hat{H} = \hat{T} + \hat{V},$$

$$\hat{T} = \sum_{i=1}^{A_1+A_2} \frac{\hat{\vec{p}}_i^{\,2}}{2m}, \quad \hat{V} = \sum_{i<j=1}^{A_1+A_2} V(\vec{r}_i - \vec{r}_j)$$

results in two-body equation of another type:

$$(\hat{T}_\rho + \hat{V}_\rho - E'\,\hat{N}_\rho)\varphi(\vec{\rho}) = 0\,,$$

$$E' = E - E_1 - E_2\,, \qquad \vec{\rho} = \frac{1}{A_1}\sum_{i=1}^{A_1}\vec{r}_i \;-\; \frac{1}{A_2}\sum_{j=A_1+1}^{A_2}\vec{r}_j\,,$$

where: $\left\langle \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \middle| \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \right\rangle = 1,\ \delta(E-E'),\ \delta(k-k'),\ etc$

$$\begin{pmatrix} \hat{N}_\rho \\ \hat{T}_\rho \\ \hat{V}_\rho \end{pmatrix} \varphi(\rho) \equiv \int \begin{pmatrix} N(\rho',\rho) \\ T(\rho',\rho) \\ V(\rho',\rho) \end{pmatrix} \varphi(\rho')\rho'^2 d\rho'; \qquad \begin{pmatrix} N(\rho',\rho'') \\ T(\rho',\rho'') \\ V(\rho',\rho'') \end{pmatrix} =$$

$$\left\langle \hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_\rho)\right\} \middle| \begin{pmatrix} \hat{1} \\ \hat{T} \\ \hat{V} \end{pmatrix} \middle| \hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho'')Y_{lm}(\Omega_\rho)\right\} \right\rangle.$$

Introducing a new WF:

$$\phi(\vec{\rho}) = \hat{N}_{\rho}^{1/2}\varphi(\vec{\rho})$$

one can obtain the Schrödinger-like equation with Hermitian Hamiltonian.

$$\left(\hat{N}_{\rho}^{-1/2}\hat{T}_{\rho}\,\hat{N}_{\rho}^{-1/2} + \hat{N}_{\rho}^{-1/2}\hat{V}_{\rho}\,\hat{N}_{\rho}^{-1/2} - E'\right)\phi(\rho) = 0\,,$$

where the habituated orthonormalization conditions take place:

$\left\langle \phi(\vec{\rho}) \middle| \phi(\vec{\rho}) \right\rangle = 1$ - for states of discrete spectra,

$\left\langle \phi_E(\vec{\rho}) \middle| \phi_{E'}(\vec{\rho}) \right\rangle = \delta(E - E'),\ etc.$ - for continuum states.

CLUSTERING IN BOUND STATES

A basic concept of the approach is the definition of measures of clustering in arbitrary A-nucleon model (cluster characteristics). Traditional definition were the following:

a) the spectroscopic amplitude –

$$C_{MDC}^{nl} = < \Psi_M \mid \hat{A}\{\Psi_D \phi_{nl}(\vec{\rho})\Psi_C\} >;$$

b) the projection of the nuclear WF onto the cluster channel – the cluster form factor (CFF) and its norm – spectroscopic factor (SF) –

$$\Phi_l(\rho) = < \Psi_M \mid \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho - \rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} >;$$

$$S_{MDC} \equiv \int \Phi(\rho)|^2 \ \rho^2 d\rho = \sum_n \left(C_{MDC}^{nl}\right)^2;$$

A great formalism was developed for calculations of the cluster characteristics :

Neudatchin VG, Smirnov YF, Nucleon Clusters in Light Nuclei. M: Nauka Moscov, 1969.
Smirnov YF, Tchuvil'sky YM, Phys. Rev. C, **15** 84 (1977).
Nemets OF, Neudachin VG, Rudchik AT, Smirnov YF, Tchuvil'sky YM, Nucleon Clusters in Atomic Nuclei and Multinucleon Transfer Reactions,
Naukova Dumka, Kiev, 1988.
Tchuvil'sky YM, Kurowsky WW, Sakharuk AA, Neudachin VG, Phys. Rev. C, 51 784 (1995).

REDEFINITION OF THE CLUSTERING MEASURES. "NEW" CLUSTER CHARACTERISTICS.

In the paper [T. Fliessbach and H.J. Mang, Nucl. Phys. A 263, 75 (1976)] the habituated view on the clustering measures was thrown doubt. The matter is that a more accurate matching procedure (point or integral) is required to deduce the amplitude and the width of a cluster channel.

$$\Phi_l(\rho) = < \Psi_M \mid \hat{A}\{\Psi_D Y_{lm}(\Omega)\Psi_C\} > = \sqrt{\frac{M!}{D!C!}} < \Psi_M \mid \Psi_D Y_{lm}(\Omega)\Psi_C >$$

$$\left(\frac{M!}{D!C!}\right)^{-1} < \hat{A}\{\Psi_D Y_{lm}(\Omega)\Psi_C\} \mid \hat{A}\{\Psi_D Y_{lm}(\Omega)\Psi_C\} > \neq 1$$

So the cluster form factors and the spectroscopic factors should be defined as:

$$F_l(r) = \left\langle \Psi_{base} \left| \hat{N}^{-1/2} \hat{A}\{\Psi_{A_1} \frac{1}{\rho^2} \delta(\rho - \rho') Y_{lm}(\Omega_{\rho'}) \Psi_{A_2}\} \right. \right\rangle.$$

where the norm (overlap) kernel takes the form:

$$N(\rho', \rho'') = \left\langle \hat{A}\{\Psi_{A_1} \frac{1}{\rho^2} \delta(\rho - \rho'') Y_{lm}(\Omega_{\rho''}) \Psi_{A_2}\} \left| \hat{A}\{\Psi_{A_1} \frac{1}{\rho^2} \delta(\rho - \rho') Y_{lm}(\Omega_{\rho'}) \Psi_{A_2}\} \right. \right\rangle.$$

It is this function should be matched with an asymptotic WF in the area where the form of “new” CFF reproduces its form

$$F_l(\rho) \leftrightarrow \phi_l(\rho)$$

while $\Phi_l \leftrightarrow \hat{N}^{1/2} \phi_l(\rho).$

$$S'_{MDC} \equiv \int | \hat{N}_\rho^{-1/2} \Phi(\rho) |^2 \ \rho^2 d\rho.$$

In the case that $\rho \to \infty;\ \hat{N} \to \hat{1}.$

Lovas RG, Liotta RJ, Insolia A, Varga K, Delion DS, Microscopic Theory of Cluster Radioactivity. Phys. Rep., 294, 265 (1998).

A term “amount of clustering” was introduced fo the Fliessbasch’s (“new”) SF.

Cluster States in Atomic Nuclei and Cluster Decay Processes. Kadmenskii SG, Kurgalin SD, Tchuvil’sky YM, Phys. Part. Nucl., **38**, 699 (2007).

CFF, SF, ASYMPTOTIC CHARACTERISTICS OF NUCLEAR STATES AND CROSS SECTIONS OF NUCLEAR REACTIONS

Calculation of CFF and SF

The cluster-channel terms of basis are built in the form:

$$\Psi_A = \frac{1}{W}\hat{A}\{\Psi_{A_1}\Psi_{A_2}\varphi_{nl}(\vec{\rho})\}_{JM_J},$$

They are not orthogonal one to another and to the shell-model components.

The basis of cluster-channel terms incorporating all channels of a certain fragmentation A1 + A2 (a complete set of internal states of each cluster) is complete. Moreover this basis is overload and even linear dependent.

The basis may be exploited itself or being added to a certain number of shell-model WF (polarization terms). In the latter case a hybrid basis appears.

The next step in shaping of a basis of general type is to build orthonormalized WFs including the cluster terms of several channels and the polarization terms. The WFs are obtained by diagonalization of the matrix

$$\left\| \begin{array}{cc} \left[1\right] & \left\langle \Psi^{(j)}_{pol} \left| \hat{A} \right| \prod_{i=1,2} \Phi^{A_i}_{N_i,L_i,M_i}(\vec{R}_i) \Psi_{A_i} \right\rangle \\ \left\langle \Psi^{(j)}_{pol} \left| \hat{A} \right| \prod_{i=1,2} \Phi^{A_i}_{N_i,L_i,M_i}(\vec{R}_i) \Psi'_{A_i} \right\rangle & \left\langle \prod_{i=1,2} \Phi^{A_i}_{N'_i,L'_i,M'_i}(\vec{R}_i) \Psi'_{A_i} \left| \hat{A}^2 \right| \prod_{i=1,2} \Phi^{A_i}_{N_i,L_i,M_i}(\vec{R}_i) \Psi_{A_i} \right\rangle \end{array} \right\| \quad (1)$$

in which the terms of the products are expressed in the form of superpositions of Slater determinants (SD).

Eigenvectors of the matrix normalized by its eigenvalues shape the desirable basis taking the form of SD linear combinations. This basis may be employed in computing of spectra of halo, clustered, resonance states and other observables.

Norm (overlap) kernel and its computation

$$\hat{N}(\rho',\rho'') = \left\langle \hat{A}\{\Psi_{A_1} \frac{1}{\rho^2}\delta(\rho-\rho'')Y_{lm}(\Omega_{\rho''})\Psi_{A_2}\} \right| \left. \hat{A}\{\Psi_{A_1} \frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_{\rho'})\Psi_{A_2}\} \right\rangle .$$

By representing of the delta function as $\sum_n |\varphi_{nl}(\rho')\rangle\langle\varphi_{nl}(\rho)|$

its eigenvalues are calculated using the formalism of so-called the cluster coefficients and take the form:

$$\varepsilon_k = < \hat{A}\{\Psi_D \; f_l^k(\vec{\rho})\,\Psi_C\} \,|\hat{1}|\, \hat{A}\{\Psi_D \; f_l^k(\vec{\rho})\,\Psi_C\} >;$$

and eigenfunctions (the result of diagonalization (1)) turn out to be expressed through the oscillator WFs

$$f_l^k(\rho) = \sum_n B_{nl}^k \phi_{nl}(\rho).$$

The final forms of CFF and SF are the following:

$$F_l(\rho) == \sum_k \varepsilon_k^{-1/2} \sum_{nk} C_{MDC}^{nl} f_l^k(\rho);$$

$$S_{AA_1A_1(l)} \equiv \int |F_l(\rho)|^2 \, \rho^2 d\rho = \sum_k \varepsilon_k^{-1} \sum_{nn'} C_{AA_1A_2}^{nl} C_{AA_1A_2}^{n'l} B_{nl}^k \; B_{n'l}^k .$$

CONFIGURATION MIXING SHELL MODEL. RESULTS AND DISCUSSION

Volya A, Tchuvil'sky YM, Phys. Rev. C **91**, 044319 (2015).

α-clustering in the ground states of (s-d)-shell nuclei

$A_P - A_D$	$S_0^{(\exp)}$ [63]	$S_0^{(\exp)}$ [64]	$S_0^{(\exp)}$ [65]	$S_0^{(\mathrm{old})}$ [24]	$S_0^{(\mathrm{old})}$ this work	$S_0^{(\mathrm{new})}$ this work
^{20}Ne-^{16}O	1.0	0.54	1	0.18	0.173	0.755
^{22}Ne-^{18}O			0.37	0.099	0.085	0.481
^{24}Mg-^{20}Ne	0.76	0.42	0.66	0.11	0.091	0.411
^{26}Mg-^{22}Ne			0.20	0.077	0.068	0.439
^{28}Si-^{24}Mg	0.37	0.20	0.33	0.076	0.080	0.526
^{30}Si-^{26}Mg			0.55	0.067	0.061	0.555
^{32}S-^{28}Si	1.05	0.55	0.45	0.090	0.082	0.911
^{34}S-^{30}Si				0.065	0.062	0.974
^{36}Ar-^{32}S				0.070	0.061	0.986
^{38}Ar-^{34}S			1.30	0.034	0.030	0.997
^{40}Ca-^{36}Ar	1.56	0.86	1.18	0.043	0.037	1

Inserting the complete set of the resonance WFs

$$1 = \sum_i |\Psi_{M_i}><\Psi_{M_i}|$$

into exp. (3) it is easy to deduce the relationship:

$$1 = \varepsilon_k^{-1} \sum_{inn'} C_{M_i DC}^{nl} C_{M_i DC}^{n'l} B_{nl}^k B_{n'l}^k$$

Performing summation over k one can obtain:

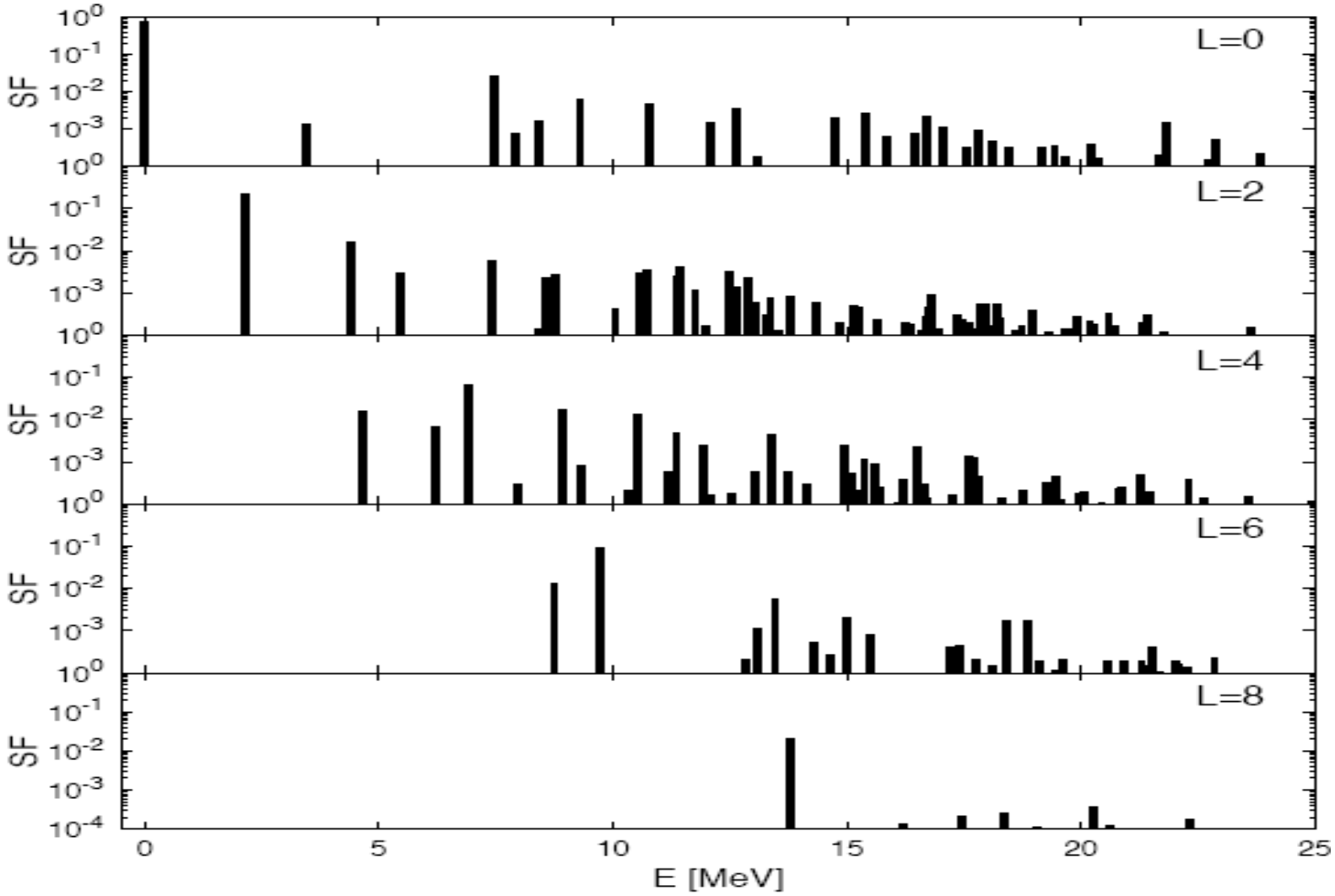
$$\sum_i S_{M_i DC}^{l'} = \dim \| k \|$$

The sum rule of the "new" spectroscopic factors corresponding to a fixed value of n (cluster strength in $2\hbar\omega$ domain turn out to be equal to unity. Thus the statistical properties are described accurately. That is critical for the dense spectra. In average:

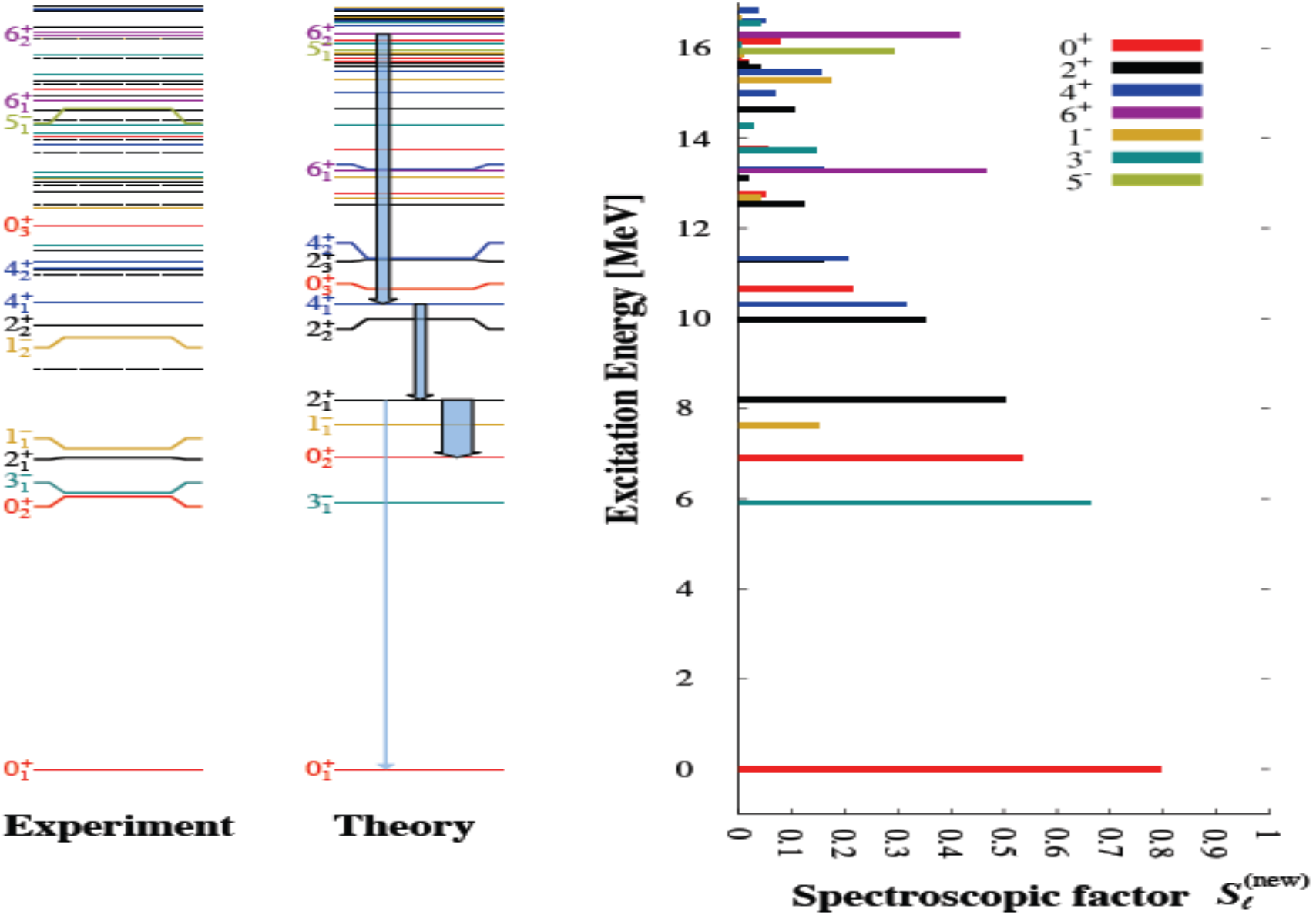
$$S_{M(E)DC}^{l} \sim \rho_l^{-1}(E)$$

GENERAL TRENDS OF THE SPECTROSCOPIC FACTORS

Spectroscopic factors of α-clusters in 32S states



α-clustering in 16O



PAULI PRINCIPLE IN HEAVY CLUSTER SYSTEMS. ARE FISSION AND FUSION CLUSTER PROCESSES?

An eigenfunction of any A-fermion Hamiltonian can be presented in the form of a superposition of determinants built up in terms of one-nucleon WFs or in terms of another basis:

$$\Psi = \sum_{N_{\min},\ldots} c_{N_{\min}\ldots} \det\{\psi_{m_i n_i}{}^{(i)}\}$$

$$N = \sum_i n_i \geq N_{\min}$$

No one transformation of the Hamiltonian and/or coordinates violates this restriction rule.

Therefore the number N_{min} is a topological invariant for fixed fermion number A. It is so-called “nodes theorem”.

And it is true for a system of bosons consisting of fermions.

The Structural Forbiddenness of the Heavy Fragmentation of the Atomic Nucleus. Smirnov YF, Tchuvil 'sky YM, Physics Letters B, 134, **25** (1984).

^{32}S example

The cluster channel WF containing the ground states of clusters

$$|\hat{A}\{\Psi_{16O}\phi_{nl}(\vec{\rho})\Psi_{16O}\}>$$

$^{32}S \quad N_{\min} = 44; \quad ^{16}O+^{16}O \quad N_{\min} = 48 \quad q = 4$

^{258}Fm symmetric fission

$$q = 48$$

In general the mass distribution of fission fragments is in correlation with the q-value.

NUCLEON AS A QUARK CLUSTER. HIDDEN COLOR

Isobaric Component of Deuteron in Quark Model Smirnov YF, Tchuvil 'sky YM, Journal of Physics G, **4**, L1 (1978)

$$\frac{1}{\Omega_N \Omega_\Delta} < \hat{A}\{\Psi_N \phi(\vec{\rho}) \Psi_N\} \,|\, \hat{A}\{\Psi_\Delta \phi(\vec{\rho}) \Psi_\Delta\} > \neq 0$$

$$\delta_\Delta = 0.3 \div 2.3\%$$

Is Deuteron a Six-Quark Model? Matveev VA, Sorba P, Nuovo Chimento Letters, **20** 435 (1977).

$$\frac{1}{\Omega_N} < \hat{A}\{\Psi_N \phi(\vec{\rho}) \Psi_N\} \,|\, \sum \frac{1}{\Omega_{c,ijk}} \hat{A}\{\Psi_{C,i} \phi_j(\vec{\rho}) \Psi_{C,k}\} >= 80\%$$

$$\delta_C = d_{[21]} d_{[21]} / d_{[222]}$$

Where $d_{[f]}$ is the dimension of the Young frame [f] – irreducible representation of the group of permutations in the color space. No the distance dependence!

CLUSTER RADIOACTIVITY

The discovery and the history

1984, H. Rose, J. Jones $^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C$

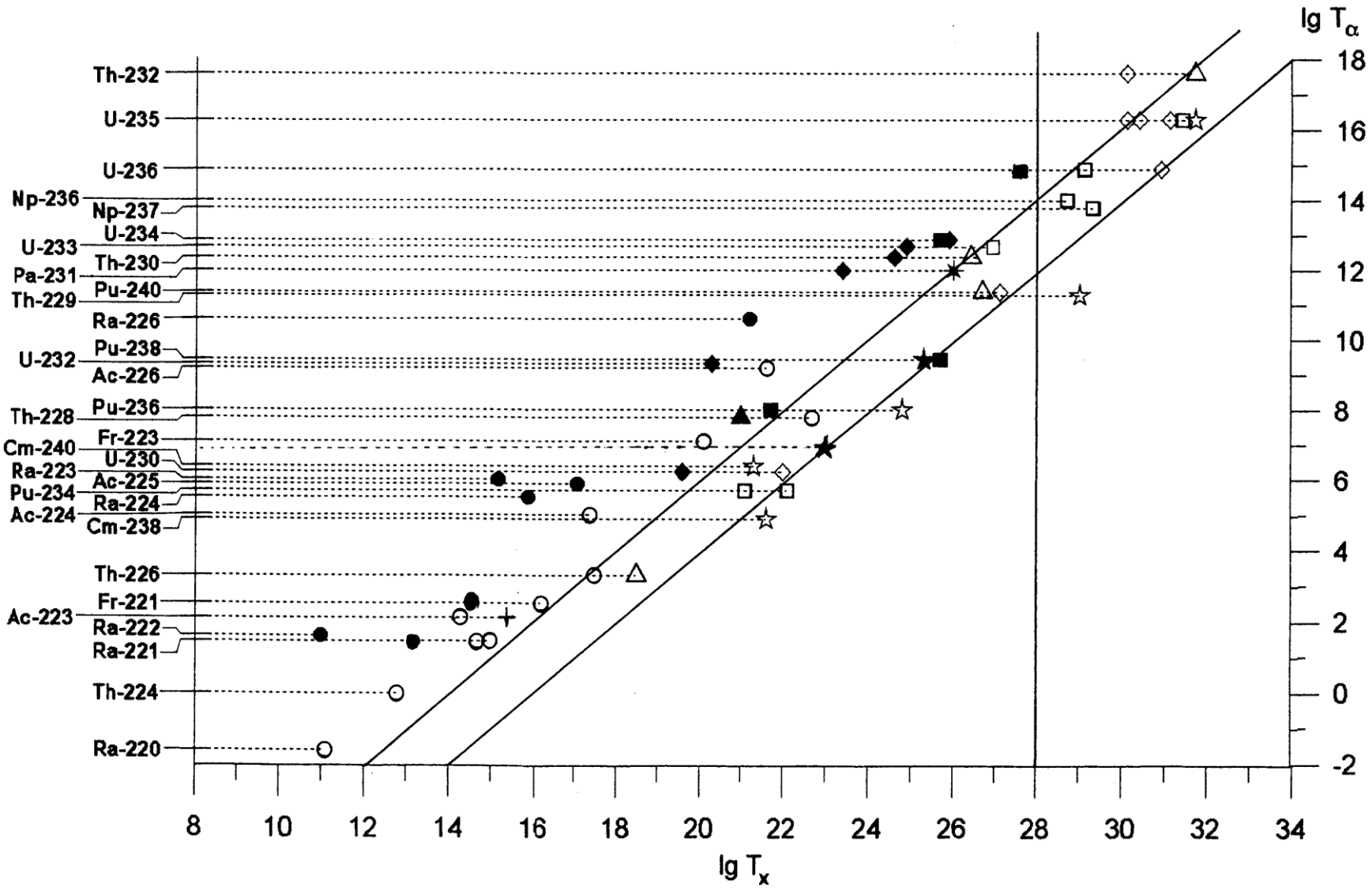
1914, E Rutherford – No other heavy particles besides alpas at the level 10^{-4}

50^{th} – 60^{th} Geochemists – too much Ar in uranium ores.

End of 70^{th} – A. Sandulescu demonstrated that the penetrability of the cluster potential barrier is of the same order as the alpha-particle one.

End of 70^{th} – beginning of 80^{th} – the group under the guide of B. Novatsky (Kurchatov Center) searched for the effect and confirmed the discovery in May 1985.

Known and promising examples



NUCLEAR CLUSTERING IN AB INITIO APPROACH

Rodkin DM, Tchuvil'sky YM, Chinese Phys. C **44** 124105 (2020); Phys. Rev. C **103**, 024304 (2021).

Is ^{8}Be a perfect cluster system?

Choice of bases is the following:

1. Conventional basis of NCSM – *mod1*.
2. Pure cluster one-channel basis (both ^{4}He clusters in GS, truncation level – $N_{max} = 0$) – *mod2*.
3. Three- or two-channel basis incorporating the realistic WFs of the first and the second 0^{+} states of ^{4}He with truncation level $N_{max} = 2$ – *mod3*.

The results are obtained by use of NN-potential **Daejeon16** which is built starting from N3LO forces (A.M. Shirokov, I.J. Shin, Y. Kim et al, PLB **761**, 87 (2016) are exploited. Code Bigstick is used for shell-model computing.

CFF and SF are computed using the procedure presented above.

The generalization procedure is the following. Right-lower sub-matrix of (1) contains WF of different states of fragments determining certain channels in this case:

$$\| N_{nn'} \| = < \Phi_{000}(\vec{R}) \Psi_{A_1} \phi_{nl}(\vec{\rho}) \Psi_{A_1} | \hat{A}^2 | \Phi_{000}(\vec{R}) \Psi'_{A_1} \phi_{n'l}(\vec{\rho}) \Psi'_{A_2} > .$$

After the diagonalization of it orthnormalized set of coupled-channel A-nucleon cluster WF appear. The sum of squared overlaps of the WF ψ_A with these WF provides a proper definition of the aggregate amount of clustering.

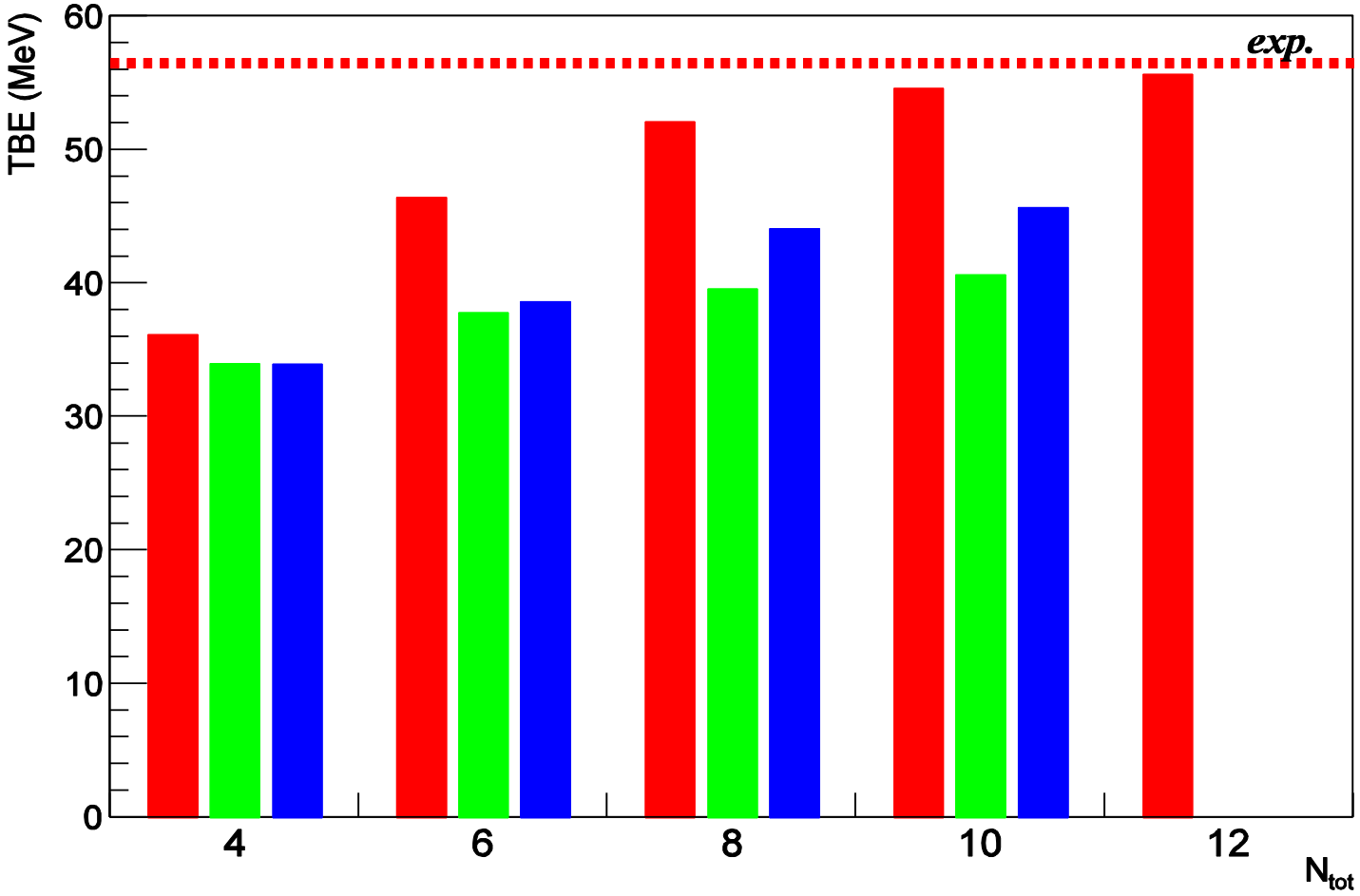
Contrary to this definition ordinary SF may be called the one-channel amount of clustering.

SPECTROSCOPIC FACTOR AND AGGREGATE AMOUNT OF CLUSTERING FOR α-PARTICLES IN ^{8}Be NUCLEUS

	N=6	N=8	N=10	N=12	N=14
α+α	0.765	0.866	0.861	0.875	0.880
α+α , α*+α	0.793	0.868	0.868		
α+α , α*+α , α*+α *	0.864	0.879	0.873		

Daejeon16 NN-potential

TOTAL BINDING ENERGIES OF ^{8}Be NUCLEUS IN VARIOUS BASES



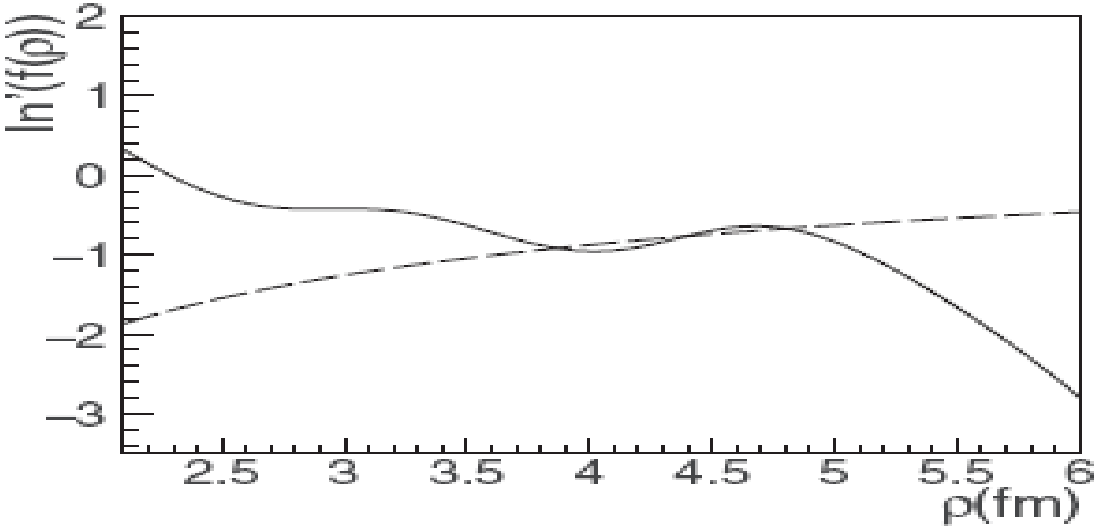
Daejeon16, ħω =15 MeV. Red – *mod1*, green – *mod2*, blue – *mod3*.

ASYMPTOTIC CHARACTERISTICS (ANC and Γ) of 8Be and 7Li NUCLEI

The asymptotic characteristics are deduced using the various R-matrix approaches.

To determine the position of the matching point R_p of the CFF and the asymptotic WF, the condition of equality of the logarithmic derivatives is used:

$$\frac{F'_l(r)}{F_l(r)} = \frac{W'_{-\eta_0,l+1/2}(2kr)}{W_{-\eta_0,l+1/2}(2kr)} \text{ or } \frac{F'_l(r)}{F_l(r)} = \frac{G'_l(\eta,r)}{G_l(\eta,r)} \qquad \frac{F_l(\eta,r)}{G_l(\eta,r)} = P_l(r_>) \ll 1$$



Logarithmic derivatives of CFF *(solid line) and* function $G_l(\rho)$ *(dashed line) for* $7/2^-$ *state of* 7Li nucleus for 4He+3H channel.

Therefore, the decay width of this resonance is given by the expression:

$$ANC = \frac{rF_l(R_p)}{W_{-\eta,l+1/2}(2kR_p)}$$

$$\Gamma_l = \frac{\hbar^2}{\mu k}\left(\frac{F_l(R_p)}{G_l(\eta,R_p)}\right)^2$$

If the resonance is wide, then the partial width is calculated using the standard R-matrix theory

$$\Gamma_\alpha = \frac{\hbar^2}{\mu k}(F_l(\eta,r)^2 + G_l(\eta,r)^2)^{-1}_{r=R_p} F_l(R_p)^2$$

For determining the decay width of subthreshold resonance, we used the formulation of (Mukhamedzhanov and Tribble,1999):

$$\Gamma_{subth}(E) = \frac{\hbar^2}{\mu_{ab}} kR_p (F_l(\eta,r)^2 + G_l(\eta,r)^2)^{-1}_{r=R_p} \frac{W^2_{-\eta_0,l+1/2}(2k_0P_p)}{R_p}\left|ANC_{ab}\right|^2$$

SPECTRA of 8Be and 7Li NUCLEI

$J^\pi, \bar{T}$	E_{bind} MeV	E^*_{dae} MeV	E^*_{exp}MeV(T)	SF	Γ_{th} MeV	Γ_{exp} MeV
0^+_1,0	56.25	0.0	0.0 (0)	0.879	7.29 eV	5.57 (6.8)* eV
2^+_1,0	52.85	3.40	3.03±0.01 (0)	0.849	1.17	1.513
4^+_1,0	44.63	11.62	11.35±0.15 (0)	0.792	2.41	3.5
0^+_2,0	44.54	**11.71**	—	**0.813**	**8.86**	—
2^+_2,0.001	42.09	**14.16**	—	**0.715**	**3.57**	—
2^+_3,0.971	39.65	16.59	16.626±0.003 (0+1)	0.0025	0.019	0.108
2^+_4,0.078	39.05	17.19	16.922±0.003 (0+1)	0.354	0.416	0.074
4^+_2,0.001	37.48	**18.76**	—	**0.288**	**3.39**	—
2^+_5,0.065	35.02	21.22	20.1±0.01 (0)	0.0459	0.434	0.8 (1.1)
0^+_3,0.852	35.01	21.23	20.2±0.01 (0)	0.0208	0.056	0.7 ($\leq$1)
0^+_4,0.315	34.44	21.80	—	0.0610	0.092	—
2^+_6,0.966	34.27	21.97	—	0.0039	0.002	—
4^+_3,0.007	34.24	22.00	19.86±0.05 (0)	0.441	5.13	0.7
2^+_7,0.028	33.57	22.70	22.2 (0)	0.059	0.135	0.8
2^+_8,0.996	33.22	23.02	—	0.001	0.004	—
0^+_5,0.017	32.91	23.33	—	0.215	1.71	—
4^+_4,0.997	32.69	23.55	—	0.0009	0.009	—

J^π	E_{expt}	E_{theor}	$E^{expt}_{\alpha+t}$	$E^{theor}_{\alpha+t}$	ANC_α [66]	ANC_α theor.	E^{expt}_n	E^{theor}_n	l	J_n	ANC_n theor.	ANC_n [78]
$3/2^-$	39.245	39.110	−2.467	−2.529	3.57 ± 0.15	3.44	−7.25	−7.639	1	1/2	−1.618	1.652
									1	3/2	1.317	1.890
$1/2^-$	38.768	38.279	−1.99	−1.69	3.0 ± 0.15	2.95	−6.77	−6.81	1	1/2	−0.531	−0.540
									1	3/2	1.979	−2.540

J^π	E_{expt}	E_{theor}	$E^{expt}_{\alpha+t}$	$E^{theor}_{\alpha+t}$	Γ_α [74]	Γ_α theor.	E^{expt}_n	E^{theor}_n	Γ_n [67]	l	J_c	$\Gamma_n(ANC_n)$ theor.	Γ theor. [42]
$7/2^-$	34.593	34.409	2.195	2.172	0.069	0.065	−2.59	−2.938		3	1/2	0.013[a]	$\Gamma_\alpha = 0.214$
$1/2^+$	32.804	28.921	3.984	7.66	3.15 [67]	7.4	−0.81 [67]	2.55	0.295 keV[b]	0	1/2	0.54 keV[b]	
$5/2^-$	32.641	31.610	4.147	4.971	0.918[c]	0.564	−0.65	−0.139		1	3/2	0.199[a]	$\Gamma_\alpha = 0.785$
$5/2^-$	31.791	30.816	4.997	5.765	0.033 [67]	0.797	0.2	0.655	0.058	1	3/2	0.053	$\Gamma_n = 0.210$
										1	1/2	0.088	
$3/2^-$	30.495	28.175	6.293	8.406	4.7[c]	0.873	1.5	3.296	0.867 [79]	1	3/2	1.0	$\Gamma_n = 1.70$
										1	1/2	0.23	
$1/2^-$	30.155	27.280	6.633	9.301	2.7[c]	0.282	1.84	4.191		1	3/2	1.0	$\Gamma_n = 2.44$
													$\Gamma_\alpha = 0.435$
$7/2^-$	29.675	28.489	7.133	8.092	0.437[c]	0.453	2.32	2.982		3	1/2	0.72 keV	$\Gamma_n = 0.039$
$3/2^-$		27.047		9.60		1.25		4.424		1	1/2	0.785	

[a] ANCs ($fm^{-1/2}$).

[b] Γ ($E_n = 1$ eV).

[c] Total decay width.

J^π	T	E_{expt}	E_{theor}	E_{He}^{expt}	E_{He}^{theor}	Γ_{tot} [74]	SF_{He}	$\Gamma_{He}(ANC_{He})$ theor.	$E_{Li^*}^{expt}$	$E_{Li^*}^{theor}$	SF_{Li^*}	$\Gamma_{Li^*}(ANC_{Li^*})$ theor.
$1/2^-$	1/2	30.155	27.280	−0.885	1.538		0.1465	0.343[a]	−1.727	0.638	0.0453	0.259[a]
$3/2^-$	3/2	28.005	27.247	1.265	1.571	0.260 ± 0.35	0.1638	0.111	0.433	0.671	0.3770	0.117

[a]ANCs ($fm^{-1/2}$).


THANK YOU FOR ATTENTION!